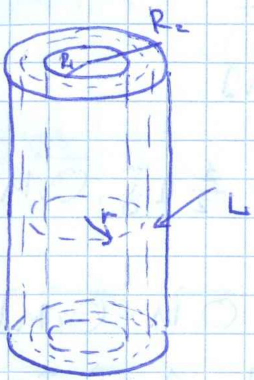
8.1

8



Воспользуемся теор. о циркуляции:

$$\oint_L \vec{H}\,d\vec{l} = H \cdot 2\pi r = \tilde{I}$$

1) $r \le R_1$: $H \cdot 2\pi r = I \cdot \dfrac{\pi r^2}{\pi R_1^2} \Rightarrow$

$$\Rightarrow H = \frac{I r}{2\pi R_1^2}$$

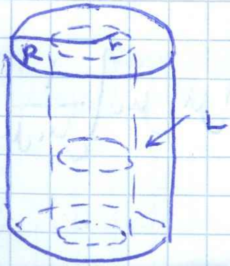
2) $R_1 \le r \le R_2$: $H \cdot 2\pi r = I \Rightarrow H = \dfrac{I}{2\pi r}$

3) $R_2 \le r$: $H \cdot 2\pi r = 0 \Rightarrow H = 0$

$$H = \begin{cases} \dfrac{I r}{2\pi R_1^2} ; & r \le R_1 \\ \dfrac{I}{2\pi r} ; & R_1 \le r \le R_2 \\ 0 ; & R_2 \le r \end{cases}$$

8.2 Рассмотрим систему из сплошных цилиндров с токами плотности j, текущ. в противоположных направлениях.

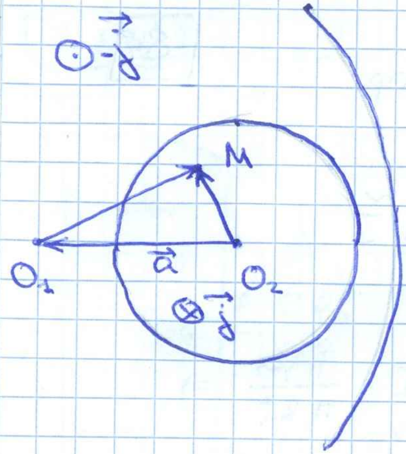
Эта система будет эквивалентна данной.



Найдём H внутри цилиндра

$$\oint_L \vec{H}\,d\vec{l} = H \cdot 2\pi r = j\pi r^2 \Rightarrow$$

$$\Rightarrow H = \frac{jr}{2}$$

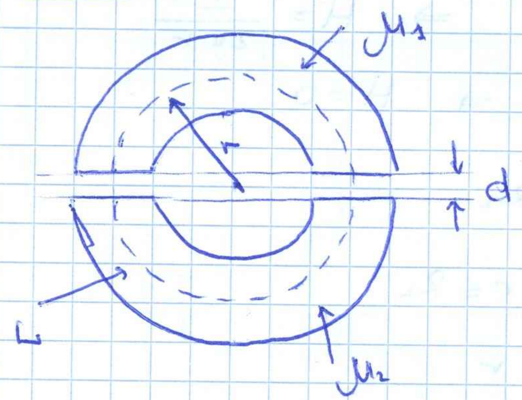


$$\vec{H}_1 = \frac{1}{2}[\vec{j}_1, \vec{r}_1] = -\frac{1}{2}[\vec{j}, \overrightarrow{O_1M}]$$

$$\vec{H}_2 = \frac{1}{2}[\vec{j}_2, \vec{r}_2] = \frac{1}{2}[\vec{j}, \overrightarrow{O_2M}]$$

$$\vec{H}_M = \vec{H}_1 + \vec{H}_2 = \frac{1}{2}[\vec{j}, \overrightarrow{O_2M} - \overrightarrow{O_1M}] = \frac{1}{2}[\vec{j}, \overrightarrow{O_2O_1}] = \frac{1}{2}[\vec{j}, \vec{a}]$$

$\boxed{8.3}$



$$\vec{H}_1 = \frac{\vec{B}(r)}{\mu_0 \mu_1}$$

$$\vec{H}_2 = \frac{\vec{B}(r)}{\mu_0 \mu_2}$$

$$\vec{H}_0 = \frac{\vec{B}(r)}{\mu_0}$$

$$\oint_L \vec{H} d\vec{l} = H_1\left(\frac{L}{2} - d\right) + H_2\left(\frac{L}{2} - d\right) + H_0 \cdot 2d =$$

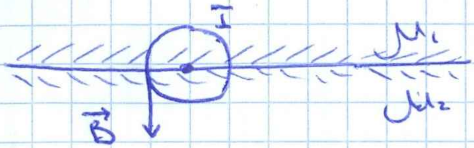
$$= \frac{B(r)}{\mu_0}\left[\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)\left(\frac{L}{2} - d\right) + 2d\right] =$$

$$= \frac{B(r)}{\mu_0}\left[(\mu_2 + \mu_1)\left(\frac{L}{2} - d\right) + 2d\mu_1\mu_2\right]\frac{1}{\mu_1\mu_2} =$$

$$= IN$$

$$B(r) = \frac{\mu_0 \mu_1 \mu_2 I N}{(\mu_1+\mu_2)\left(\frac{L}{2}-d\right)+2d\mu_1\mu_2}$$

$\boxed{8.4}$



$$B_{1n} = B_{2n} = B$$

$$\mu_1\mu_0 H_1 = \mu_2\mu_0 H_2 = B$$

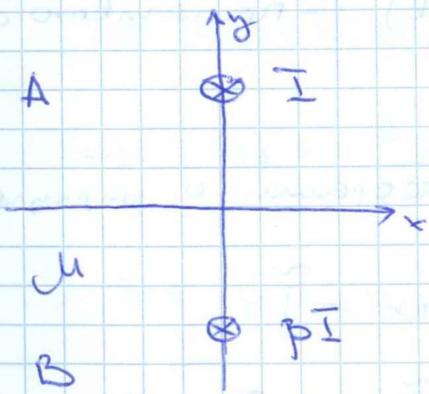
$$\mu_1 H_1 = \mu_2 H_2$$

$$I = \oint H dl = \pi r \cdot H_1 + \pi r \cdot H_2 = \pi r (H_1 + H_2)$$

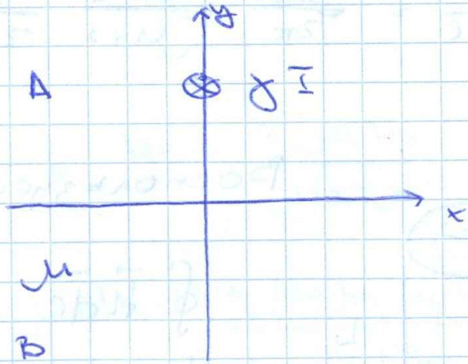
$$I = \pi r \left(\frac{\mu_2 H_2}{\mu_1} + H_2\right) \Rightarrow \frac{\pi r}{\mu_1}(\mu_2 H_2 + \mu_1 H_2)$$

$$H_2 = \frac{I\mu_1}{\pi r(\mu_1+\mu_2)} \Rightarrow B = \frac{\mu_1\mu_2 I \mu_0}{\pi r(\mu_1+\mu_2)}$$

$\boxed{8.7}$ Воспользуемся методом изображений



для определения поля в A



для определения поля в B

$$H = \frac{I}{2\pi r} \sim I$$

$$\begin{cases} H_{\tau_1} = H_{\tau_2} \\ H_{n_1} = \mu H_{n_2} \end{cases} \Rightarrow \begin{cases} \beta - 1 = -\gamma \\ \beta + 1 = \mu\gamma \end{cases}$$

$$1 - \gamma + 1 = \mu\gamma \Rightarrow \gamma = \frac{2}{\mu+1}$$

$$\beta = 1 - \gamma \Rightarrow \quad \Rightarrow \beta = \frac{\mu - 1}{\mu + 1}$$

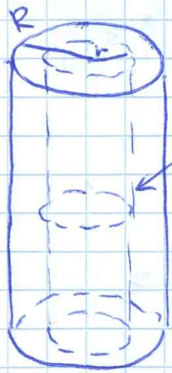
т. о.

$$\vec{dF} = I\,[\vec{dl}, \vec{B}_\beta] = I \cdot dl \cdot \frac{\beta I \mu_0}{2\pi \cdot 2h}(-\vec{k}) =$$

$$= \frac{\mu_0}{2\pi} \cdot \frac{I^2}{2h} \cdot \frac{\mu - 1}{\mu + 1}\, dl\, (-\vec{k})$$

$$\frac{\vec{dF}}{dl} = \frac{\mu_0}{2\pi}\, \frac{\mu - 1}{\mu + 1}\, \frac{I^2}{2h}\, (-\vec{k})$$ – притягиваются.

$\boxed{8.8}$



Воспользуемся теоремой о циркуляции

$$\oint_L \vec{H}\,\vec{dl} = H \cdot 2\pi r = \tilde{I}$$

1) $r \le R$ $\quad H = \frac{\tilde{I}}{2\pi r} = \frac{I}{2\pi r} \cdot \frac{\pi r^2}{\pi R^2} =$

$$= \frac{I r}{2\pi R^2}$$

2) $r \ge R$ $\quad H = \frac{I}{2\pi r}$

$$B = \mu_0 \tilde{\mu} H = \begin{cases} \mu \mu_0 \dfrac{Ir}{2\pi R^2}, & r \le R \\ \mu_0 \dfrac{I}{2\pi r}; & r \ge R \end{cases}$$

$\boxed{8.5}$ Очевидно, что для контура $H_{\tau_1} = H_{\tau_2} = 0$.

$$H_1 \mu_1 = H_2 \mu_2 \Rightarrow \quad H_2 = \frac{\mu_1}{\mu_2} H_1$$

Поле контура соленоидально и симм. относительно границы раздела:

$$\oint_L H_0 \, dl = I \qquad (1)$$

$$\oint_L \tilde{H} \, dl = \oint_{L_1} H_1 \, dl + \oint_{L_2} H_2 \, dl = I \quad (2)$$

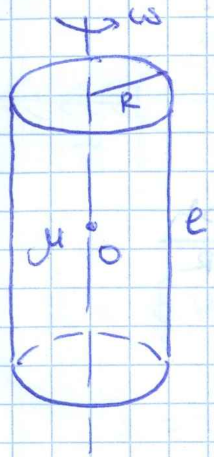
$$L_1 = L_2 \; ; \; L_1 \cup L_2 = L$$

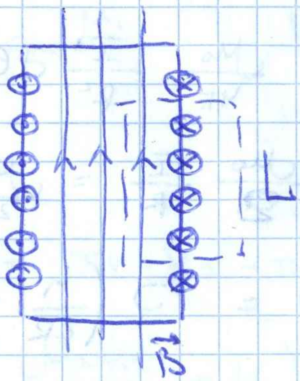
$$\Rightarrow \; 2H_0 = H_1 + H_2$$

$$\Rightarrow \quad H_1 = \frac{2\mu_2 H_0}{\mu_2 + \mu_1} \; ; \quad H_2 = \frac{2\mu_1 H_0}{\mu_2 + \mu_1}$$

$$B = \mu_1 \mu_0 H_1 = \mu_2 \mu_0 H_2 = \frac{2\mu_0 \mu_1 \mu_2 H_0}{\mu_1 + \mu_2}$$

8.13





Цилиндр тонкий и длинный

$\Downarrow$

$\ell \ll R$

$I = \frac{Q}{2\pi}\omega$

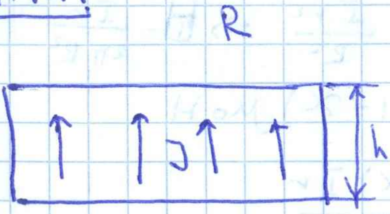
Поле внутри соленоида почти однородно.

Поле вне соленоида (вблизи его) почти отсутствует.

$$\oint \vec{H}\,d\vec{\ell} = \frac{B}{\mu\mu_0}L = \frac{Q}{2\pi}\omega$$

$$B(0) = \frac{\mu\mu_0 Q\omega}{2\pi L}; \quad H = \frac{B}{\mu\mu_0} = \frac{Q\omega}{2\pi L}$$

8.14



т.к. намагниченность однородна, то внутри магнита токов нет

$\Downarrow$

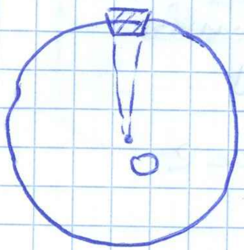
только на поверхности

из гранич. условий $J_{2\tau} - \underset{0}{J_{1\tau}} = i'$

$J_{2\tau} = J$

$\Rightarrow i' = J$

Диск тонкий ⇒ можно считать кольцом радиуса R, с током $I' = hi'$.

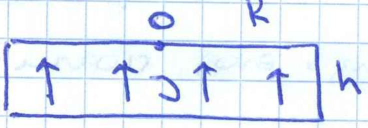


$$dB = \frac{\mu_0}{4\pi}\frac{I'}{R^2}\,dl$$

$$B = \frac{\mu_0}{2}\frac{I'}{R} = \frac{\mu_0}{2}\frac{Jh}{R}$$

$$H = \frac{B}{\mu_0} - J = J\left(\frac{h}{2R} - 1\right)$$

$\boxed{8.11}$

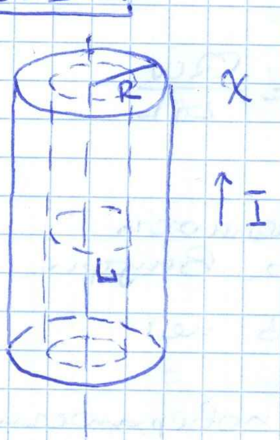


Диск тонкий ⇒

⇒ можно считать кольцом.

$$(O):\quad B_{n_1} = B_{n_2} = B_1 = B_2 = \frac{\mu_0}{2}\frac{Jh}{R} \quad (8.14)$$

$\boxed{8.15}$



1) $0 \le r \le R$

$$\oint_L H\,dl = \frac{I}{\pi R^2}\pi r^2 = \frac{I r^2}{R^2}$$

$$2\pi r \cdot H = \frac{I r^2}{R^2} \Rightarrow H = \frac{I r}{2\pi R^2}$$

$$B = \mu\mu_0 H = (1+\chi)\mu_0 H$$

$$B = \frac{\mu_0(1+\chi) I r}{2\pi R^2}$$

2) $R \le r$ $\quad \oint_L H\,dl = I \Rightarrow 2\pi r \cdot H = I \Rightarrow H = \frac{I}{2\pi r}$

$$B = \mu_0 H = \mu_0 \frac{I}{2\pi r}$$

$\boxed{8.12}$

$$\begin{cases} J = \alpha z + \beta \\ J(0) = J_m \\ J(\ell) = 0 \end{cases} \Rightarrow J(z) = \frac{J_m}{\ell}(\ell - z)$$

$$J(z) = i(z) \Rightarrow dI = i(z)\,dz$$

$$dB = \frac{\mu_0 dI}{2} \frac{R^2}{(R^2+z^2)^{3/2}} = \frac{\mu_0 J(z)}{2} \frac{R^2\,dz}{(R^2+z^2)^{3/2}} =$$

$$B = \int_0^\ell dB = \underbrace{\int_0^\ell \frac{\mu_0 J_m}{2} \frac{R^2}{(R^2+z^2)^{3/2}} dz}_{I_1} - \underbrace{\int_0^\ell \frac{\mu_0 J_m}{2\ell} \frac{R^2 z\,dz}{(R^2+z^2)^{3/2}}}_{I_2}$$

$$I_1 = \frac{\mu_0 J_m}{2} \int_0^\ell \frac{R^2}{(R^2+z^2)^{3/2}} dz = \frac{\mu_0 J_m}{2} \frac{\ell}{(R^2+\ell^2)^{3/2}}$$

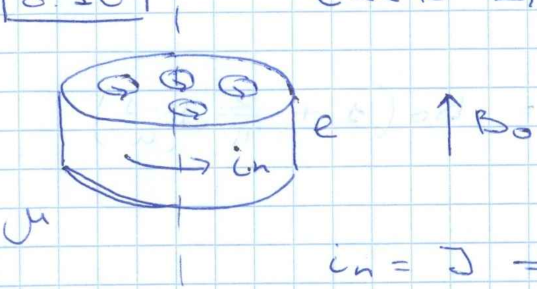
$$I_2 = \frac{\mu_0 J_m R^2}{2\ell} \int_0^\ell \frac{z\,dz}{(R^2+z^2)^{3/2}} = \frac{\mu_0 J_m R^2}{2\ell}\left[\frac{-1}{\sqrt{R^2+z^2}}\right]\Bigg|_0^\ell =$$

$$= \frac{\mu_0 J_m R^2}{2\ell}\left(\frac{1}{R} - \frac{1}{(R^2+\ell^2)^{1/2}}\right)$$

$$B = \frac{\mu_0 J_m}{2} \frac{\ell}{(R^2+\ell^2)^{3/2}} + \frac{\mu_0 J_m}{2}\left(\frac{R}{\ell} - \frac{R^2}{\ell(R^2+\ell^2)^{1/2}}\right) =$$

$$= \frac{\mu_0 J_m}{2\ell}\left(\sqrt{R^2+\ell^2} - R\right) \approx \frac{\mu_0 J_m}{2}(1 - R/\ell)$$

$\boxed{8.10}$ $l \ll R \Rightarrow$ считаем кольцом



$$B = \mu\mu_0 H = (1+\chi)\mu_0 H = B_0 + \underbrace{\mu_0 \chi H}_{B'}$$

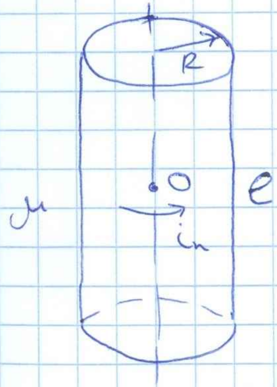
$$i_n = J \Rightarrow I = i_n l = J l$$

$$B' = \frac{\mu_0 I}{2R} = \frac{\mu_0 J l}{2R} \;(=)$$

$$J = \chi H = (\mu - 1)\frac{B_0}{\mu\mu_0}$$

$$(=)\; \frac{\mu-1}{\mu}\,\frac{B_0 l}{2R} \Rightarrow B = B_0\left(1 + \frac{\mu-1}{\mu}\,\frac{l}{2R}\right)$$

$\boxed{8.9}$ $R \ll l$



$$B = \mu\mu_0 H = (1+\chi)\mu_0 H = B_0 + \underbrace{\mu_0 \chi H}_{B'}$$

$$i_n = J \Rightarrow I = i_n l = J l$$

$$dB' = \frac{\mu_0}{2\pi}\,\frac{dI \cdot R^2}{(R^2+z^2)^{3/2}} = \frac{\mu_0}{2\pi}\,\frac{R^2 J\,dz}{(R^2+z^2)^{3/2}}$$

$$B' = \int_{-l/2}^{l/2} \frac{\mu_0}{2\pi}\,\frac{J R^2}{(R^2+z^2)^{3/2}}\,dz =$$

$$= \frac{\mu_0 J}{2\pi}\left[\frac{z}{(z^2+R^2)^{1/2}}\right]\Bigg|_{-l/2}^{l/2} = \frac{\mu_0 J}{2\pi}\,\frac{l}{(l^2/4+R^2)^{1/2}} =$$

$$= \frac{\mu_0 J}{2\pi}\,4 = \frac{2\mu_0 J}{\pi}$$

$$J = \chi H = (\mu - 1)\frac{B_0}{\mu\mu_0}$$

$$B' = \frac{2}{\pi}\,\frac{\mu-1}{\mu}\,B_0 \Rightarrow B = B_0\left(1 + \frac{2}{\pi}\,\frac{\mu-1}{\mu}\right)$$